# Examples

This addendum provides a detailed description of the equilibrium strategies for several known games. Each example is followed by an informal discussion, highlighting the main features of its spe. By no means is this discussion aimed at providing a complete characterization of the equilibrium of each game. It does, however, provide many more details than those mentioned in the text. We also hope that the set of the examples would provide additional intuition for the way the model works.

In all examples, for ease of exposition we use $C_i(a_i \to a_i', t) = c(t)$ for any $i$, $a_i$, $a_i'$. The results would (qualitatively) be the same if we had a general cost structure satisfying the conditions given in Section 2. In all examples the row player is denoted by player 1, and the column player by player 2, and all parameters are positive.

## Battle of the Sexes

| | *Boxing* | *Opera* |
|---|---|---|
| *Boxing* | $A, b$ | $0, 0$ |
| *Opera* | $0, 0$ | $a, B$ |

where $A > a$, $B > b$.

- The equilibrium outcome is: [*Boxing,Boxing*] if $b > a$, [*Opera,Opera*] if $a > b$. If $a = b$ the equilibrium depends on the grid, even for fine grids.

- Key equilibrium stages: off equilibrium, the two players play a war of attrition at profile [*Boxing,Opera*], i.e. at each player's favorite activity. If, for example, $b < a$, there exists a stage of the game that lasts from $c^{-1}(b)$ to $c^{-1}(a)$, in which player 2 is fully committed to playing Opera for the rest of the game, while player 1 is still flexible. At this stage, player 1 prefers to switch to Opera, giving him positive payoffs of $a - c(t)$. Both players foresee this, resulting in player 1 giving up immediately.

- Comments: First, note that it does not matter how much one likes his favorite activity, but what matters is how much one likes his least favorite activity. The player who wins is the one who likes it less, making the threat of "staying at home" credible. Second, note that if $a = b$ the game is not grid invariant–the player who wins is the player who can commit first before the common critical point $t^* = c^{-1}(a) = c^{-1}(b)$, i.e. player $i$ wins if and only if $prev_i(t^*) < prev_j(t^*)$.

## Prisoners' Dilemma

| | *Defect* | *Coop* |
|---|---|---|
| *Defect* | $d, d$ | $D, c$ |
| *Coop* | $c, D$ | $C, C$ |

where: $D > C > d > c$.

- The equilibrium outcome is: [*Defect,Defect*] if $(D-C) \geq (d-c)$, and [*Coop,Coop*] if $(d-c) > (D-C)$.

- Key equilibrium stages: clearly, each player can guarantee at least $d$ by defecting all the time. Hence, we need to check if [*Coop,Coop*] can be supported as an equilibrium. Suppose first that $(D - C) \geq$

$(d-c)$. In such a case, there exist a point in time on the grid, in which at least one of the players (both, in case the inequality is strict) would defect, knowing that the switching costs would be too high for the opponent to react by defecting as well. Both players know it, and hence begin by defecting. Once $(d-c) > (D-C)$ such a point in time does not exist, and [*Coop,Coop*] can be supported in equilibrium. In this case, as long as it is potentially profitable for a player to defect, it is profitable for the other player to retaliate by defecting as well.

- Comments: Note that in this case the common critical time does not matter for the equilibrium outcome. For example, if $(d-c) > (D-C)$ one can easily verify that the switches that become active at the same time are "unrelated"–one's origin is neither the origin nor the destination of the other.

## Generalized Matching Pennies

| | $A$ | $B$ |
|---|---|---|
| $A$ | $K, \epsilon_1$ | $0, m$ |
| $B$ | $0, m$ | $k, \epsilon_2$ |

where: $\epsilon_1, \epsilon_2 \approx 0$, $K > m$, $K \geq k$.

- The equilibrium outcome is: [*A*,*A*] if $\epsilon_1 > \epsilon_2$, [*B*,*B*] if $\epsilon_2 > \epsilon_1$. However, if $k < m - \epsilon_1$ and $\epsilon_1 > \epsilon_2$ then play begins with [*B*,*A*] and is switched (by player 1) to [*A*,*A*] only after $t = c^{-1}(m - \epsilon_1)$. If $\epsilon_1 = \epsilon_2$ then player 2 is indifferent, and both outcomes are equilibria.

- Key equilibrium stages: it is clear that player 1 wins the game because he is the most flexible player, allowing him to enjoy the "second-mover" advantage. After $t = c^{-1}(m - \epsilon_1)$, player 1 can still react, while for player 2 it is already too costly. Still, player 2 can at least guarantee his maxmin payoffs by playing $A$ if $\epsilon_1 > \epsilon_2$, and $B$ otherwise. Thus, even though $K > k$, player 1 cannot obtain $K$ for sure. Moreover, when $k < m - \epsilon_1$ and $\epsilon_1 > \epsilon_2$, even though player 2 plays $A$, player 1 must delay in equilibrium. This is because there is a stage of the game, between $c^{-1}(k)$ and $c^{-1}(m - \epsilon_1)$, in which player 2 would switch to $B$ at profile [*A*,*A*], knowing that it will not be contested by player 1. In order to avoid this, player 1 starts by playing $B$, and switches to $A$ only at $c^{-1}(m - \epsilon_1)$, once such a switch by player 2 is not profitable any longer.

- Comments: In the fully symmetric case, in which $\epsilon_1 = \epsilon_2 = 0$ and $K = k = m$, the grid matters. The player who wins is the player who moves last before the common critical point $t^* = c^{-1}(m) = c^{-1}(K)$, i.e. player $i$ wins if and only if $prev_i(t^*) > prev_j(t^*)$.

## Delay with Super-Dominant Strategy

| | $L$ | $R$ |
|---|---|---|
| $U$ | $13, 3$ | $1, 10$ |
| $D$ | $0, 5$ | $0, 0$ |

- This is the game analyzed in Section 3.5 of the paper. Here we provide a more complete description of the equilibrium. Note that action $U$ is super-dominant for player 1 (see definition in Section 3.5).

- The equilibrium path is to play [*D*,*L*] until $t = c^{-1}(7)$, and then switch (by player 1) to [*U*,*L*].

- Key equilibrium stages: the key stage is between $c^{-1}(3)$ and $c^{-1}(7)$. During this stage, at profile $[U,L]$ player 2 finds it profitable to switch to $R$. After $c^{-1}(7)$, the switching costs are greater than the benefits of the switch $(10 - 3 = 7)$, thus the switch is not profitable anymore. Before $c^{-1}(3)$, a switch by player 2 would allow player 1 to credibly switch to $D$. This switch would be followed by player 2 switching back to $L$, taking the game back to its equilibrium path, which ends at $[U,L]$. Therefore, player 1 avoids this off-equilibrium switch by player 2 by playing first $D$, and switching to $U$ only after $c^{-1}(7)$. This gives him payoffs of $13 - c(c^{-1}(7)) = 6$, which are more than 1 (what he would have obtained if he had played $U$ to begin with). Note also that this is credible only because player 1 can commit to stick to $U$ had player 2 played $R$, because 3 is greater than 1 (there is an off-equilibrium war of attrition at the early stage of the game at the profile $[D,R]$).

- Comments: the key point is that late enough in the game, but still early for player 2 to react, player 1 can commit himself not to play his super-dominant strategy as long as player 2 does not "cooperate". If this commitment was not attainable (for example, if the payoffs of player 1 at $[U,R]$ were greater than 3), player 2 could simply play $R$, knowing that player 1 would eventually switch to his super-dominant strategy.