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Labour Supply and Hours Constraints

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1 INTRODUCTION

Most empirical labour supply studies rely on the assumption that an individual can freely choose her hours of work, at her (given) market wage rate. This stands in contrast with responses of individuals in many surveys. For example, in Ham (1982) a table compiled from the Michigan University Panel Study of Income Dynamics (PSID) shows that between the years 1967 and 1974 around 9–14 per cent of males claimed to be underemployed. Using this information Ham (1982) proceeds to test the hypothesis that no individual is constrained, and rejects it.

While survey responses alone may not form reliable evidence, the existing econometric evidence on constraints among both workers and non-workers seems strong enough to warrant further consideration. Thus in this paper we exploit the switching regressions model to introduce demand side variables in the determination of observed hours of work, while separately identifying the parameters determining desired labour supply. Hence we allow for a non-zero probability of being constrained while working.

The switching regressions model was discussed by Quandt (1972, 1982) and estimation by maximum likelihood and methods of moments has been discussed by Kiefer (1978) and Quandt and Ramsey (1978). Recent developments, relating to error specification and testing for normality using contaminated normal distributions, can be found in Arellano and Bover (1986). Here we extend the basic switching regressions model to allow both truncated and censored samples. We use this specification to generalize the standard labour supply model to allow for constraints in the hours worked by workers. To validate our results we develop and implement a set of

We should like to thank Richard Blundell, Olympia Bover, Andrew Chesher, John Ham, Geert Ridder and Ian Walker as well as participants of the Microeconometrics conference at Toulouse University for helpful comments and discussion. The comments of an anonymous referee greatly improved the paper. All remaining errors and shortcomings are those of the authors. Finance for this research was provided by the Economic and Social Research Council under project B20022060.

diagnostic tests for these models. These are adaptations of the normality and heteroscedasticity tests presented by Blundell and Meghir (1986), Bera and Jarque (1982) or Lee (1984).

The empirical results obtained with our model are compared with those obtained assuming that workers can always choose their hours of work (given their wage rate). We find that constraints are important among certain groups of workers. Furthermore we find that the estimated labour supply parameters can be quite sensitive to the assumption of no constraints.

The paper is organized as follows. In section 2 we present models that are appropriate in the absence of any information relating to constraints. In section 3 we discuss normality and heteroscedasticity tests for the truncated and censored switching regressions model. We present our empirical results in section 4 and in section 5 we offer some concluding remarks and discuss avenues for further research.

2 THE BASIC MODELS

Blundell et al (1987) have developed a double hurdle model for labour supply where the observations are assumed to be generated by the scheme

$$h_i = \begin{cases} h_i^* & \text{if } h_i^* > 0 \text{ and } Q_i > 0 \\ 0 & \text{otherwise} \end{cases} \tag{9.1}$$

where h_i are the abserved hours of work, h_i^* are the individual's desired hours of work and Q_i is a random variable determining whether the worker can get a job or not. Model (9.1) allows for search unemployment at zero hours of work. The individuals are separated into three groups: those working their desired hours of work, those not willing to work at their (given) market wage rate and those not working but willing to work some positive number of hours. This specification, while very useful for analysing unemployment using cross-section data, cannot capture other plausible possibilities in the labour market.

Firstly, some non-working individuals may accept a job offer implying non-optimal hours as long as this alternative is preferred to unemployment. In this study we assume that hourly wages, given demand conditions, are fixed and relate to the individual's labour market experience. Thus the wage offer distribution is assumed to be degenerate. The individual may then obtain offers relating to the number of hours worked per week at the given personal wage rate.

Secondly, individuals already working a particular number of hours, that may have been optimal when chosen, may want to change these hours, in response to changed economic or demographic conditions, e.g. a change in the marginal tax rate. If hours cannot be varied at the current job, the individual will start searching for another job without necessarily quitting the current one.

Thirdly, demand side shocks may lead the employer to require an increase or decrease in the hours worked, at the same hourly wage rate. If the wage does remain fixed the individual will find herself at a non-optimal position. She may then start to search for a new job without necessarily quitting the current one. (For a discussion on the job search, see Burdett and Mortensen (1978).)

With perfect job mobility and adequate labour market flexibility we would always observe individuals at their optimal position, but in the presence of search costs or other constraining factors we cannot reasonably expect immediate adjustment. So we assume that the population can be partitioned into two groups: those who are constrained in their choices and consequently are off their labour supply curve, and those who are unconstrained. This type of assumption groups together the overemployed and the underemployed workers. While this may introduce some mis-specification, in view of the data available for this study, and because of lack of any straightforward identifying restrictions to separate the underemployed from the overemployed, we did not attempt to estimate a model separating these two groups.

We postulate that, associated with each individual, there is a positive probability of being in each of the two regimes. The probability of being unconstrained is specified to be

$$P(D_i > 0) = P(z_i'\delta + e_i > 0) = \Phi_i \tag{9.2}$$

where $D_i = z_i'\delta + e_i$ and z_i is a vector of observable variables, including individual characteristics and market conditions. The sign of D_i determines whether an individual is constrained or not. The error term e_i is assumed to be N(0, 1) so that $\Phi_i = \Phi(z_i'\delta)$ where $\Phi(\ )$ represents the cumulative normal distribution.

The observed hours h_i of the individual belonging to the unconstrained subpopulation can be described by the standard labour supply model derived from utility maximization. If the wage offer distribution is degenerate and in the absence of fixed costs, h_i can be represented by means of a Tobit equation:[1]

$$h_i = \max(h_i^*, 0) \tag{9.3}$$

with

$$h_i^* = g(w_i, x_i; \theta) + u_i \tag{9.4}$$

where w_i is the marginal after-tax wage and x_i is a vector of individual-specific characteristics including non-labour income y_i: θ is an unknown vector of parameters and u_i is an error term summarizing unobservable characteristics and tastes. We assume that $u_i \sim \mathrm{N}(0, \sigma_u^2)$.

In contrast, for constrained individuals (9.4) becomes irrelevant as a model of observed hours of work. In this case we assume that the constrained outcome d_i can be described by some function of demand side and individual-specific characteristics:

$$d_i = r_i'\alpha + v_i \tag{9.5}$$

Positive d_is are not equivalent to observed hours of work since the job offer will be declined if the utility obtained as a non-participant is larger than at d_i. Defining $C_i = \log\{U_i(d_i)/U_i(0)\}$ where $U_i(d_i)$ is the utility of accepting the offered package (implying constrained hours) and $U_i(0)$ is the utility of remaining unemployed, for the constrained individuals we can write

$$h_i = \begin{cases} d_i \text{ if } C_i > 0 \\ 0 \end{cases} \tag{9.6}$$

where $C_i > 0$ indicates that d_i is preferred to unemployment. Noting that $d_i \leqslant 0$ implies $C_i \leqslant 0$, then $P(d_i > 0 \mid C_i > 0,\ D_i < 0) = 1$. Consequently the single censoring rule $C_i > 0$ is sufficient for this regime. Equation (9.5) is a reduced form equation which could result from complicated interactions between demand and supply side effects. Accordingly we could specify a reduced form indicator function for C_i depending on both individual and demand side characteristics given the dependence of C_i on the offered hours d_i. This would lead to a highly overparameterized model which in practice would be difficult to identify, particularly using the truncated sample (workers only). Thus, for the purposes of this empirical study, we chose to approximate $P(C_i > 0)$ with $P(d_i > 0)$, although for expositional purposes we use the formal model implied by (9.6).

The probability of observing a non-worker as implied by (9.3) and (9.6) is

$$\begin{aligned} P(h_i = 0) = 1 &- P(D_i > 0)P(h_i^* > 0 \mid D_i > 0) \\ &- P(D_i < 0)P(C_i > 0 \mid D_i < 0) \end{aligned} \tag{9.7}$$

since

$$P(d_i > 0 \text{ and } C_i > 0 \mid D_i < 0) = P(d_i > 0 \mid C_i > 0,\ D_i < 0)P(C_i > 0 \mid D_i < 0)$$

and

$$P(d_i > 0 \mid C_i > 0,\ D_i < 0) = 1.$$

Then the density function for the positive observations becomes

$$\begin{aligned} f(h_i \mid w_i, x_i, r_i, z_i; \theta, \sigma_u, \alpha, \sigma_v, \delta) &= \Phi_i f^1(h_i \mid w_i, x_i; \theta, \sigma_u) \\ &+ (1 - \Phi_i) f^2(h_i \mid r_i; \alpha, C_i > 0, \sigma_v) P(C_i > 0) \end{aligned} \tag{9.8}$$

where $f^1(\ )$ and $f^2(\ )$ are the density functions relating to (9.4) and (9.5) respectively.

Combining (9.7) and (9.8) we can construct the sample likelihood function. Yet there are many reasons for not wanting to use the observations relating to the *non-workers*. Firstly, wage rates for the *unemployed* are not observed and have to be imputed or integrated out. Secondly, information relating to the occupation and skills of the non-worker are often not available and have to be proxied by other variables. Selecting a sample of workers only (and adjusting for selection bias) overcomes the above difficulties. Thus we first present a log likelihood function for the sample of workers only (truncated sample). This takes the form

$$L_1 = \sum_{+} \{\log(\Phi_i f_i^1 + (1 - \Phi_i) f_i^2 F_i^2) - \log(\Phi_i F_i^1 + (1 - \Phi_i) F_i^2)\} \quad (9.9)$$

where Σ_+ is the summation for positive hours of work, $F_i^1 = P(h_i^* > 0)$ and $F_i^2 = P(C_i > 0)$.

The sample log likelihood function (9.9) is a truncated version of the standard switching regressions likelihood function. A distinguishing feature of our approach is that we allow the probability of each regime to vary across the sample. This seems quite important in our context as the presence of constraints is likely to be affected by personal characteristics, such as education or variables capturing the conditions of regional labour markets.

The model presented above uses only the subsample relating to working women. The advantages of such an approach were dicussed earlier. A main argument was that non-workers do not report an hourly wage rate. Consider as an alternative the case where the wage rate can be explained by a wage equation of the form

$$\log(w_i) = \beta' q_i + \tau_i \quad (9.10)$$

where $\tau_i \sim N(0, \sigma_\tau^2)$. We could maximize the joint likelihood function for $\log(w_i)$ and h_i (observed hours) over the whole sample. In principle this would overcome the problem of missing wages for the non-participants, but given a nonlinear model for labour supply such an aproach seems computationally intractable. Thus, in our empirical work, when using the entire sample we simply impute the wages for the non-workers using the expectations given by a wage equation of the form (9.10). The resulting censored sample log likelihood function takes the form

$$L_2 = \sum_{+} [\log\{\Phi_i f_i^1 + (1 - \Phi_i) f_i^2 F_i^2\}] + \sum_{0} [\log\{\Phi_i (1 - F_i^1) + (1 - \Phi_i)(1 - F_i^2)\}] \quad (9.11)$$

where F_i^1 and F_i^2 are defined as above.

The likelihood function (9.11) reveals certain interesting features and combines alternative approaches found in the literature. Suppose that we had information separating the individuals into those who were constrained and those who were not. We could then estimate a model over the latter subsample after conditioning on this selection. This is equivalent to using only those terms in L_2 that are weighted by Φ_i, over the appropriate subsample. The parameters in Φ_i which would determine the selection correction could be identified using a probit over constrained and unconstrained individuals. This approach is similar to that used by Ham (1982). Now note that the parts of L_2 weighted by $1 - \Phi_i$ (the probability of being constrained) consist of a probit describing the probability of accepting a suboptimal job offer weighted by the density function of such job offers.

Finally note that implicit in L_2 are a set of cross-equation restrictions. This can best be seen by noting that F_i^2 is the probability that offered hours d_i are preferred to unemployment. Consequently a full utility specification of our model would imply cross-equation restrictions between the hours density function f_i^1 and the distribution function F_i^1 (which is the integral of f_i^1) and F_i^2. Thus (9.11) is a tightly specified likelihood function. Here to simplify the computations we do not impose all the structural restrictions and in fact we assume, as mentioned earlier, that $P(C_i > 0) = P(d_i > 0)$. If we apply this approximation the truncated version of our likelihood function becomes

$$L_1' = \sum_{+} [\log\{\Phi_i f_i^1 + (1-\Phi_i) f_i^2\} - \log\{\Phi_i F_i^1 + (1-\Phi_i) F_i^2\}] \quad (9.9')$$

where $F_i^2 = P(d_i > 0)$.[2] Similarly the censored likelihood function becomes

$$L_2' = \sum_{+} [\log\{\Phi_i f_i^1 + (1-\Phi_i) f_i^2\}] + \sum_{0} [\log\{\Phi_i(1-F_i^1) + (1-\Phi_i)(1-F_i^2)\}] \quad (9.11')$$

Given a sufficiently general specification for d_i, including individual and demand side characteristics, we believe that our simplifying approximation will not bias the results severely.

The likelihood function (9.11) generalizes both the double hurdle and the Tobit. If $\Phi_i = 1$ for all observations then we obtain the standard Tobit model for labour supply (see for example Layard et al., 1980). In such a model all individuals are assumed to be unconstrained. If, however, $F_i^2 = 0$ then we obtain the double hurdle. The latter implies that all working individuals are on their labour supply functions but some of the non-workers are willing to work and cannot get a job. Finally the truncated version of (9.11), i.e. (9.9), nests the standard truncated model and hence nests the labour supply model with no hours constraints.

Unfortunately, in all the above cases the likelihood ratio test statistic (LR) does not have a χ^2 distribution. This is because certain parameters are not identified under the null hypothesis. Consider first of all an LR test between the Tobit model and (9.11). The parameters defining f_i^2 are not identified under the hypothesis that the Tobit model is correct. Alternatively, if the null hypothesis model is the double hurdle the parameters α in (9.5) are not identified. A similar problem arises with (9.8) versus the truncated regression model. In such situations the LR test statistic is not χ^2, except conditional on some fixed values for the non-identified parameters. This problem was originally discussed by Davies (1977) and is further discussed by Arellano and Bover (1986). Thus the likelihood comparisons that we make in the empirical section of the paper must be interpreted as informal diagnostics.

3 DIAGNOSTIC TESTS

In this section we discuss the Lagrange multiplier (score) tests for non-normality and heteroscedasticity for truncated and censored switching regressions models. We follow the approach of Bera and Jarque (1982) to derive statistics based on the Pearson family of distributions. For clarity we base our presentation on the censored model. The results for the truncated model are then derived with ease.

To derive the non-normality test for each of the two errors in the equations we replace each of the densities in (9.9′) or (9.11′) by a general representation of the Pearson family of densities (see Kendall and Stuart, 1977). This family comprises densities satisfying the differential equation

$$\mathrm{d}\log c(u) = \frac{c_1 - u}{c_0 - c_1 u - c_2 u^2}\,\mathrm{d}u \tag{9.12}$$

where u is a random variable and the c_j are parameters of the distribution. In particular the normal distribution is obtained by setting $c_1 = c_2 = 0$. The parameter c_0 then represents the variance of the random variable u. Using (9.12) the mixture of densities becomes

$$f(u) = \Phi k_1 \exp\left\{\int \mathrm{d}\log c(u)\right\} + (1-\Phi)k_2 \exp\left\{\int \mathrm{d}\log s(u)\right\} \tag{9.13}$$

where $c(u)$ and $s(u)$ are two members of the Pearson family, as defined in (9.12), with parameters c_j and s_j respectively and Φ is the mixture probability. The normalizing constants k_1 and k_2 are functions of c_j and s_j ($j = 0, 1, 2$), respectively. A non-normality test would consist of the joint hypothesis that $c_j = s_j = 0$ for $j = 1, 2$. In what follows we derive the scores for c_j. Those for s_j can be derived similarly.

In general, the scores for testing the normality assumption underlying our switching regressions model can be derived by noting that

$$\frac{\partial \log f}{\partial c_j} = \Phi \frac{f^1}{f}\frac{\partial \log f^1}{\partial c_j} \tag{9.14}$$

where f^1 is the normal density function corresponding to the first regime. Hence $\partial \log \mathrm{f}^1/\partial c_j$ is the score relating to the standard model, where all observations are generated by the first regime. Following the methodology discussed in Gouriéroux et al. (1987) the scores for the corresponding censored model can be written as

$$I(h>0)\Phi\frac{f^1}{f}\frac{\partial \log f^1}{\partial c_j} + \{1 - I(h>0)\}\Phi E_{f1}\left(\frac{\partial \log f^1}{\partial c_j}\bigg| h<0\right) \tag{9.15}$$

where E_{f1} is the expectation with respect to the density function f^1 and

$I(h>0)=1$ if $h>0$ and 0 otherwise. In (9.15) we have used the fact that

$$E_{f1}\left(\frac{\partial \log f^1}{\partial c_j}\middle| h<0\right)=E_f\left(\frac{f^1}{f}\frac{\partial \log f^1}{\partial c_j}\middle| h<0\right). \tag{9.16}$$

The expressions for the conditional expectations are well known and are given in appendix 9A.

The scores relevant to the truncated model can be derived now by noting that the likelihood function for a truncated sample can be written as the difference of a censored sample likelihood function and that of a discrete choice (probit) model. Hence, the scores take the form

$$\Phi\left\{\frac{f^1}{f}\frac{\partial \log f^1}{\partial c_j}-E_{f1}\left(\frac{\partial \log f^1}{\partial c_j}\middle| h>0\right)\right\}. \tag{9.17}$$

To construct a score relevant for testing heteroscedasticity we specify as an alternative

$$c_{i0}=c_0(1+l'm_i)^2 \tag{9.18}$$

and similarly for s_{i0}. In (9.18) m_i is some vector of explanatory variables and l is a vector of parameters. Under homoscedasticity $l=0$.

To compute the value of the Lagrange multiplier (score) test statistic we use the R^2 from a regression of ones on the matrix of derivatives of the alternative likelihood function (with respect to all parameters). These derivatives are evaluated at the point that maximizes the likelihood function under the null. The value of the test statistic then is simply HR^2 where H is the sample size (see Chesher, 1983).

4 EMPIRICAL RESULTS

Past empirical studies of labour supply have emphasized the importance of a general non-additive preference structure (see for example Blundell and Meghir, 1986, and the literature cited there). This is particularly important in the context of this paper, where mis-specification in the labour supply equation (9.4) could increase the scope of the alternative equation (9.5), thus leading to a spurious rejection of the standard labour supply model. A convenient choice for (9.4) is the quadratic labour supply function[3]

$$h_i^*=\alpha(x_i)+\beta(x_i)w_i+\delta(x_i)y_i+\theta(x_i)w_i^2+u_i \tag{9.19}$$

where x_i is a vector of demographic and taste shifter variables, w_i is the hourly marginal wage rate (after accounting for the tax and benefit system) and y_i is 'other income', calculated using the budget constraint, i.e. $y_i=c_i-w_ih_i$, c_i being the observed household consumption. In using this measure of 'other income' we ensure that the labour supply model is consistent with intertemporal two-stage budgeting. In this way we account for possible intertemporal substitution (see Blundell, 1986; Meghir, 1985). The properties of (9.19) are described in detail by Stern (1986). Moreover we specify

$$\alpha(x_i) = \alpha^0 + \alpha^1 n_1 + \alpha^2 n_2 + \alpha^3 n_3 + \alpha^4 n_4 + \alpha^{\mathrm{Age}}\mathrm{Age} + \alpha^{\mathrm{Age2}}\mathrm{Age}^2 + \alpha^{\mathrm{Ed}}\mathrm{Education} \tag{9.20a}$$

$$\beta(x_i) = \beta^0 + \beta^1 n_1 + \beta^2 n_2 + \beta^3 n_3 + \beta^4 n_4 \tag{9.20b}$$

$$\delta(x_i) = \delta^0 + \delta^1 n_1 + \delta^2 n_2 + \delta^3 n_3 + \delta^4 n_4 \tag{9.20c}$$

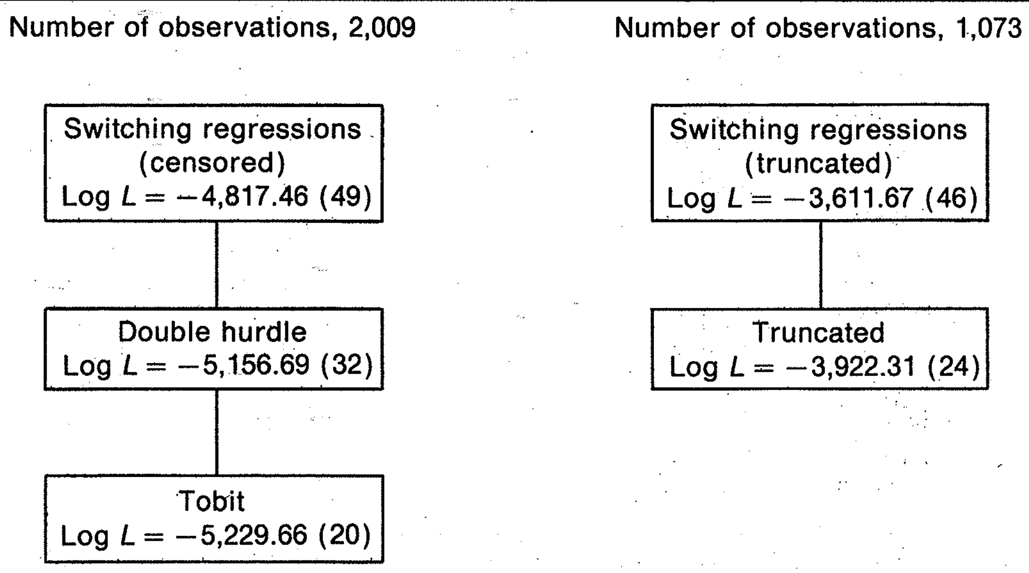
$$\theta(x_i) = \theta^0 + \theta^1 n_1 + \theta^2 n_2 + \theta^3 n_3 + \theta^4 n_4 \tag{9.20d}$$

where Age is the female age, Education is the school-leaving age and n_i are the number of children in the jth age group (0–2, 3–5, 6–10 and 11 +). Thus our specification allows for both flexible wage responses and a wide variety of interactions between taste shifter variables on the one hand and income and wage on the other.

The empirical results presented in this section relate to a sample of 2,009 married women drawn from the UK Family Expenditure Survey for 1981. A simple description of the data used as well as a glossary for variable definitions is included in appendix 9B.

We begin the presentation of our results with table 9.1, first column, where the values of the maximized log likelihood functions for a nested sequence of models are shown.[4] At the bottom of the column is the most restrictive model (Tobit), which assumes that both employed and unemployed individuals are unconstrained. The LR test statistic between this model and the double hurdle, which allows for constraints among the unemployed, is 145.94. This

Table 9.1 Likelihoods for nested sequences of models



Log L is the value of the maximized log likelihood function. The total number of estimated parameters is given in parentheses.

provides clear evidence against the Tobit model. The LR test statistic between the censored switching regressions model and the double hurdle is 678.46. Although this test statistic does not have a known distribution, because of the problems discussed in section 2, its magnitude does seem to suggest rejection of the hypothesis of no constraints (among workers as well as among non-workers). A similar informal comparison is presented between the standard truncated model and the switching regressions truncated model in table 9.1, second column.

Given these preliminary results we present estimates for the competing models using data on workers only. In this way we avoid relying on imputed wage rates for the unemployed, and more importantly we can use the female occupational data which are not available for non-workers.[5] The competing models now are the standard truncated regression model and the truncated switching regressions model (9.9′).

In table 9.2 parameter estimates relating to the standard truncated model are presented. The general conclusions are similar to those obtained in earlier work by Blundell and Meghir (1986). Labour supply seems to be backward bending for most demographic groups. Education has a strong positive effect on hours, while age has a strong negative effect. Interestingly, the hypothesis of a linear labour supply would not be rejected by this specification as the θ parameters are not (at least individually) significantly different from zero. The normality test statistics seem quite reasonable, although the heteroscedasticity test is clearly very large, pointing to some remaining mis-specification.

Table 9.2 Labour supply parameters (truncated model)

	Parameter	*Standard error*		*Parameter*	*Standard error*
α^0	36.82891	6.14315	δ^0	−0.08295	0.01352
α^1	4.15469	6.55254	δ^1	−0.09199	0.06292
α^2	7.39000	5.34068	δ^2	−0.15722	0.05230
α^3	0.25235	2.58928	δ^3	0.01514	0.01592
α^4	1.84571	2.75360	δ^4	−0.01196	0.01333
α^{Ed}	5.55275	1.98735	θ^0	−0.25116	0.91162
α^{Age}	−4.17181	2.69866	θ^1	1.52755	1.00995
α^{Age2}	0.27509	0.34419	θ^2	−0.42473	0.86439
β^0	−1.03991	3.26182	θ^3	0.98550	0.90015
β^1	−11.23106	5.33943	θ^4	0.54776	0.95077
β^2	−3.62944	4.10679	σ_u	10.09108	0.24514
β^3	−6.70302	2.99091			
β^4	−1.93928	3.31904			

Log *L*, −3,922.31; kurtosis (1), 4.34; skewness (1), 4.18; heteroscedasticity (3), 88.95 (variables included in this and following heteroscedasticity tests are age, education and children less than ten years).
Number of observations, 1,073.
All test statistics shown in this and subsequent tables are asymptotically χ^2 with degrees of freedom shown in parentheses.

Table 9.3 Labor supply parameters (switching regressions model)

	Parameter	*Standard error*		*Parameter*	*Standard error*
α^0	43.45955	2.37275	δ^0	−0.00222	0.00391
α^1	−25.68434	3.96877	δ^1	−0.67650	0.04332
α^2	3.19085	3.52167	δ^2	−0.07366.	0.01424
α^3	−0.38280	1.32589	δ^3	−0.00473	0.00898
α^4	−2.62095	1.35590	δ^4	0.00517	0.00861
α^{Ed}	−2.39472	0.72881	θ^0	1.00636	0.63968
α^{Age}	1.22578	0.91313	θ^1	−0.54809	0.56518
α^{Age2}	−0.14336	0.12061	θ^2	−2.56386	0.90091
β^0	−4.53479	2.21122	θ^3	0.35854	0.25871
β^1	26.49800	4.15540	θ^4	−2.11295	0.57562
β^2	2.66600	3.70900	σ_u	1.50445	0.12694
β^3	0.17598	0.01245			
β^4	4.87245	1.71760			

Log L, −3,611.67; skewness (1), 10.90; kurtosis (1), 1.39; heteroscedasticity (3), 15.60. Number of observations, 1,073

In table 9.3 the parameter estimates for the labour supply equation, as obtained by maximizing (9.9′), are shown. It is immediately clear that these estimates are quite different from those of table 9.2. Firstly, although labour supply seems to be backward bending at a large number of sample points, the result is not as strong as the standard truncated model suggests. For example a childless woman has a labour supply function that is forward sloping for wages larger than £2.25. Moreover, women with young children have a forward-sloping labour supply function over the entire range of positive marginal wage rates. Secondly, the income effects are in general smaller. Finally the hypothesis of a linear labour supply function is now rejected. In fact the labour supply parameters are much better determined in this model than in the standard truncated model presented in table 9.2.

The diagnostic tests at the bottom of table 9.3 are overall quite acceptable, although the skewness test statistic is rather large. The heteroscedasticity test statistic is now greatly reduced. The LR test statistic between the truncated model and the switching regressions model is 621.28, suggesting a rejection of the former.

The parameter estimates for equation (9.5) are presented in table 9.4. These seem quite reasonable overall. For example, vacancies (although not very significant) tend to increase the hours of the constrained workers, while overall unemployment in the industry (Fiu) tends to reduce them. Female age has a strong negative effect (a quadratic term was not significant). It is interesting to note from the labour supply parameters in table 9.3 that age has a negative effect on desired labour supply only after the age of 43. The overall strong negative correlation between hours and age, picked up by the truncated model, seems to include demand side effects. In (9.4) we have also included two dummies to capture demographic composition as well as

Table 9.4 Parameters for the constrained regime

	Parameter	*Standard error*
Intercept	35.72569	4.90874
D_1	−11.37309	1.72692
D_2	−5.08634	0.96668
Manual	−5.50781	0.91834
Services	−5.30656	2.92381
Vacif	7.36788	4.67978
Redif	0.98080	0.27309
Education	−0.36870	2.23904
Age	−0.91163	0.46280
Gross wage	−0.06713	0.19795
Fiu	−0.53829	0.18508
Income	−9.45077	1.47038
σ_v/σ_u	5.96642	0.55922

Skewness (1), 0.927; kurtosis (1), 101.4; heteroscedasticity (3), 1.23; join test of normality and heteroscedasticity (10), 134.2.

income. The first dummy D_1 is equal to unity when a young child (less than 3) is present. The second dummy D_2 is equal to unity when children between the ages of 3 and 18 are present, provided that $D_1 = 0$. All three of these variables have the expected negative sign. The most difficult variable to interpret is the occupational dummy 'Manual' (which is unity when the woman is a manual worker). It has a strong negative effect on hours, and probably reflects demand side conditions not captured by the demand side variables Vacif, Redif and Fiu.

The last parameter in table 9.4 is the ratio of the standard deviations of the error terms of (9.4) and (9.5) (u_i and v_i respectively). This parameterization was chosen in case a constrained optimization technique, coupled with a grid search, would have been necessary to identify the consistent root of the likelihood function. Such a technique would have been necessary if the maximization algorithm had entered an area of the parameter space where the switching regressions likelihood function is unbounded (see Kiefer, 1978). It turned out that in practice standard Newton–Raphson techniques were sufficient.

The diagnostic tests for this equation are presented at the end of table 9.4. Both the skewness and the heteroscedasticity test statistic are very small. Yet a worrying aspect of the results is the large kurtosis test statistic. This is probably due to some remaining mis-specification in equation (9.5) which may not be capturing the full complexity of the way in which job offers interact with personal preferences. A possible remedy would be to obtain better demand side variables. Additional regional demand side variables as well as more detailed demographics were tried but dit not lead to significant improvements.

Finally we turn to the parameter estimates for the constrained probability index (9.2) (table 9.5). Here again the results are quite plausible and suggest that the probability of facing constraints is highly variable over the sample. Manual workers (in 1981) have a much greater probability of being constrained, as do older workers. Moreover, greater unemployment in the industry leads to a higher probability of being constrained. An interesting result is the positive sign of the redundancy variable Redif. The interpretation of this coefficient, conditional on overall unemployment in the industry, is not completely clear. It may be that redundancies imply some restructuring in the industry, which in turn gives greater opportunities to the workers that remain employed to choose their hours of work. In order to improve comprehension of these results we present in table 9.6 sample averages, over the sample of workers, of the probability of being unconstrained. Each number outside parentheses represents the average probability of being unconstrained, given that the individual belongs to the specified cell in the table. The number in parentheses represents the proportion of individuals represented by each cell. Thus 78 per cent of young non-manual workers are predicted to be unconstrained. At the other extreme, only 31 per cent of older manual workers are predicted to be unconstrained.

Overall, the results obtained from this switching regressions model seem quite interesting and have offered a framework for analysing the effect of exogenous demand side variables on labour supply. One important conclusion is that demand side variables such as unemployment and vacancies have a direct effect on hours of work. Clearly this research is not complete and better

Table 9.5 Constraint probability index

	Parameter	*Standard error*
Intercept	1.77496	0.73515
Manual	−0.89160	0.14956
Services	−0.26270	0.44092
Vacif	−1.05585	0.75133
Redif	0.14323	0.04282
London	−0.08804	0.18793
Education	0.10816	0.33630
Age	−0.26733	0.06471
Fiu	−0.09928	0.04460

Table 9.6 Probabilities of being unconstrained by age and occupation

	Age below 36.9 years	*Age above 36.9 years*
Manual	0.46 (0.26)	0.31 (0.24)
Non-manual	0.78 (0.31)	0.61 (0.19)

Average probability over all the sample, 0.55.
Number of observations, 1,073.

data relating to demand side variables as well as some sample separation information would improve the analysis and allow further interpretation.

5 CONCLUDING REMARKS

In this paper we consider possible estimation techniques for the parameters of the labour supply functions when some individuals are constrained in their choice of hours of work. Moreover an informal test for the absence of constraints is obtained. As no sample separation information is available, we allow for a probability that each individual in the sample is constrained. This probability, as well as the function determining hours for the constrained individuals, depends on demand side and individual specific characteristics.

In order to present some evidence as to the statistical reliability of our empirical results we develop normality and homoscedasticity tests for the censored and switching regressions model. These turn out to be very similar to those of the standard truncated and censored models.

Our empirical results relate to a sample of married women drawn from the UK Family Expenditure Survey for 1981. A quadratic labour supply function is used which allows for flexibility in labour supply responses and is linear in parameters. The empirical results do show evidence of constraints among workers. Demand side variables were found to affect significantly the probability of being constrained. Most of the diagnostic tests were reasonable in size. The exception is the kurtosis test for the equation determining the hours of the constrained individuals. We believe that an improvement in the quality of the demand side variables should cure this.

Appendix 9A Derivation of scores for the diagnostic tests

The derivatives of the log of the mixture (9.13) with respect to c_0, c_1 and c_2, evaluated under the null hypothesis of normality, are (we have dropped the observation subscript for convenience)

$$\begin{aligned}
\frac{\partial \log f}{\partial c_0} &= \frac{\Phi f^1}{2\sigma_u^2 f}\left(\frac{u^2}{\sigma_u^2} - 1\right) \\
\frac{\partial \log f}{\partial c_1} &= \frac{\Phi f^1}{f}\left(\frac{u}{\sigma_u^2} - \frac{u^3}{3\sigma_u^4}\right) \qquad (9A.1) \\
\frac{\partial \log f}{\partial c_2} &= -\frac{\Phi f^1}{f}\left(\frac{u^4}{4\sigma_u^4} - \frac{3}{4}\right)
\end{aligned}$$

where f^1 is defined in (9.8). The scores in (9A.1) form the basis of the non-normality and heteroscedasticity test. The score for these parameters (under the null) is

$$\frac{\partial \log f}{\partial l}=\frac{\Phi f^1}{2\sigma_u^2 f}\left(\frac{u^2}{\sigma_u^2}-1\right)m \tag{9A.2}$$

where m is a vector of exogenous variables, varying across observations. To derive the scores for the censored and truncated versions of the switching regressions model we derive the expectations of (9A.1) conditional on the relevant selection rule (worker or non-worker). For the workers these conditional expectations are

$$\begin{aligned}
E\left(\frac{\partial \log f}{\partial c_0}\middle| h>0\right) &= \frac{\Phi}{2\sigma_u^2 \Pr(h>0)}\int\left(\frac{u^2}{\sigma_u^2}-1\right)f^1\,\mathrm{d}u \\
&= \frac{\Phi}{2\sigma_u^2}\frac{u^{(2)}}{\sigma_u^2} \\
E\left(\frac{\partial \lg f}{\partial c_1}\middle| h>0\right) &= \frac{\Phi}{\Pr(h>0)}\int\left(\frac{u}{\sigma_u^2}-\frac{u^3}{3\sigma_u^4}\right)f^1\,du \\
&= \Phi\left(\frac{u^{(1)}}{\sigma_u^2}-\frac{u^{(3)}}{3\sigma_u^4}\right) \\
E\left(\frac{\partial \log f}{\partial c_2}\middle| h>0\right) &= -\frac{\Phi}{\Pr(h>0)}\int\left(\frac{u^4}{4\sigma_u^4}-\frac{3}{4}\right)f^1\,du \\
&= -\frac{\Phi}{4\sigma_u^4}u^{(4)}
\end{aligned} \tag{9A.3}$$

where the $u^{(j)}$ are residual-like quantities for the censored or grouped data model (see Chesher and Irish, 1987). These generalized residuals take the form

$$\begin{aligned}
u^{(1)} &= \sigma_u \mu \\
u^{(2)} &= -g\sigma_u \mu \\
u^{(3)} &= 2\sigma_u^3 \mu + g^2 \sigma_u \mu \\
u^{(4)} &= -3\sigma_u^3 \mu g\left(\frac{1+g^2}{3\sigma_u^2}\right).
\end{aligned} \tag{9A.4}$$

where μ is the hazard function defined by $\mu=\phi(g/\sigma_u)/\Pr(h>0)$ and $\phi(\ )$ is the standard normal density function. The function $g=g(\ )$ is the deterministic part of the labour supply equation and is defined by equation (9.4).

The scores for the truncated and censored switching regressions model can be constructed using expressions (9A.1), (9A.3) and (9A.4). For the truncated model these have the general form (see Gouriéroux et al., 1987)

$$\frac{\partial \log L^A}{\partial c_j}=\frac{\partial \log f}{\partial c_j}-E\left(\frac{\partial \log f}{\partial c_j}\middle| h>0\right) \tag{9A.5}$$

The derivative (9A.5) is evaluated under the null. For the censored model the scores for the positive observations consist of the first expression on the right-hand side of (9A.5). For the observations relating to zero hours of work the scores consist of the expectations of the expressions in (9A.1) conditional on observed hours of work being zero. These scores are similar in form to those defined in (9A.3).

Appendix 9B

The data for the empirical study were drawn from the UK Family Expenditure Survey 1981. The following sample selection was applied.

1 Occupation of workers: professional, teachers, clerical, shop assistants, manual workers (skilled, semi-skilled and unskilled), i.e. FES variable A210 with value 2, 3, 4, 5, 6, 7, 8.
2 Age of adults: women, $16 < \text{age} < 60$; men, $16 < \text{age} < 65$.
3 Two adult households with the two adults a couple.

Glossary of Variable Definitions and Abbreviations

n_1	number of children less than 2 years of age
n_2	number of children aged 2 or more but less than 5
n_3	number of children aged 5 or more but less than 11
n_4	number of children aged 11 or more
D_1	1 if $n_1 > 0$ and 0 otherwise
D_2	1 if $n_2 > 0$ and $n_1 = 0$ and 0 otherwise
D_3	1 if $n_3 > 0$ and $n_1 = n_2 = 0$ and 0 otherwise
D_4	1 if $n_4 > 0$ and $n_1 = n_2 = n_3 = 0$ and 0 otherwise
Age	age of wife
Education	age of wife on leaving education
Male employed	1 if husband employed
w	female marginal wage rate
Gross Wage	female pre-tax hourly wage rate
y	unearned income computed from the budget identity $c - wh$, where c is household consumption
Fau	female unemployment rate by age
Fiu	unemployment rate by (female) industry
Vacir	vacancies registered by region/1,000
Redir	redundancies registered by region/1,000
Vacif	vacancies registered by industry (female)/1,000
Redif	redundancies registered by industry (female)/1,000
London	1 if resident in London
Manual	1 if occupation is manual (female)
Services	1 if the female works in a services-related industry

Data Analysis

Variable	Mean	Standard deviation
n_1	0.255	0.517
n_2	0.151	0.380
n_3	0.491	0.758
n_4	0.425	0.753
Age	35.890	10.750
Education	15.619	2.081
Female hours (workers only)	25.937	11.835
y	39.292	34.682
w_f (workers only)	1.363	1.207
Gross wage	2.024	2.362

Notes

1 However, as pointed out by a referee, under more general conditions there is no reason why the censoring mechanism should be geared by the number of hours, even in the unconstrained regime. In this case a generalized selectivity model would be an appropriate specification for the first regime, but given the focus of this paper we did not pursue this extension.

2 Note that in (9.9) $f_i^2 F_i^2 = f(d_i | C_i > 0) P(C_i > 0)$. Applying our approximation amounts to replacing this by $f(d_i | d_i > 0) P(d_i > 0) = f(d_i)$.

3 In earlier work on this subject the full quadratic specification was used, including the squared income term as well as the interaction between the wage and income. As these additional terms were found to be completely insignificant in all circumstances, they were dropped from the specification.

4 These tests use the entire sample including the non-workers. For the latter, wages were imputed from an estimated wage equation. Rather than imputing the occupational characteristics of the non-working females, we used male characteristics. Thus for example instead of using unemployment in the female's industry (which is not observed for non-workers) the equivalent variable for the husband was used. These variables are bound to be less informative than the actual variables, which are used in subsequent parts of the paper when we concentrate on workers only.

5 The parameter estimates and the standard errors for the censored model are available from the authors on request.

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