Robust Priors for Average Marginal Effects: Comment

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On page 511 of Arellano and Bonhomme (2009a) the formula for the bias B_M of average marginal effects in Theorem 5 should be as follows.

$$\begin{aligned} B_M &= \left[\operatorname*{plim}_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \frac{\partial}{\partial\theta}\Big|_{\theta_0} m_i(\theta,\alpha_{i0})\right] B \\ &\quad + \operatorname*{plim}_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \frac{1}{\pi_i(\alpha_{i0}|\theta_0)} \frac{\partial}{\partial\alpha}\Big|_{\alpha_{i0}} \left[\mathbb{E}_{\theta_0,\alpha}\left(-v_i^{\alpha_i}(\theta_0,\alpha)\right)\right]^{-1} \pi_i(\alpha|\theta_0) m_i^{\alpha_i}(\theta_0,\alpha). \end{aligned}$$

The difference with the formula in the paper is that the expectation $\mathbb{E}_{\theta_0,\alpha}$ is relative to the distribution of the data indexed at the α with respect to which the derivative is taken. Similarly, on p. 512 in (28) $\mathbb{E}_{\theta_0,\alpha_{i0}}\left(-v_i^{\alpha_i}(\theta_0,\alpha_i)\right)$ should be replaced by $\mathbb{E}_{\theta_0,\alpha_i}\left(-v_i^{\alpha_i}(\theta_0,\alpha_i)\right)$, and (29) should be

$$\pi_i^{R,m}(\alpha_i|\theta) = \frac{m_i^{\alpha_i}(\theta_0,\overline{\alpha}_i(\theta))}{m_i^{\alpha_i}(\theta_0,\alpha_i)} \frac{\mathbb{E}_{\theta_0,\alpha_i}\left[-v_i^{\alpha_i}(\theta_0,\alpha_i)\right]\mathbb{E}_{\theta_0,\alpha_{i0}}\left[-v_i^{\alpha_i}(\theta_0,\overline{\alpha}_i(\theta))\right]}{\mathbb{E}_{\theta_0,\overline{\alpha}_i(\theta)}\left[-v_i^{\alpha_i}(\theta_0,\overline{\alpha}_i(\theta))\right]\left\{\mathbb{E}_{\theta_0,\alpha_{i0}}\left[v_i^2(\theta_0,\overline{\alpha}_i(\theta))\right]\right\}^{\frac{1}{2}}}.$$

Derivation. On p. 7 of the supplementary appendix (Arellano and Bonhomme, 2009b) the final expression for $m_i(\theta_0,\widehat{\alpha}_i(\theta_0))$ should include the additional $O_p(1/T)$ term

$$m_i^{\alpha_i}(\theta_0,\alpha_{i0})\left(\left[-v_i^{\alpha_i}(\theta_0,\alpha_{i0})\right]^{-1} - H^{-1}\right) v_i(\theta_0,\alpha_{i0}),$$

with a similar addition to the expression for $\widehat{M}_i(\theta_0)$ on p. 8 (and in Lemma S2 on p. 7).

On p. 9, the expression for $\widetilde{B}_M$ should thus be

$$\begin{aligned} \widetilde{B}_M &= \operatorname*{plim}_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} m_i^{\alpha_i}(\theta_0,\alpha_{i0}) T\mathbb{E}_{\theta_0,\alpha_{i0}}\left(\left[-v_i^{\alpha_i}(\theta_0,\alpha_{i0})\right]^{-1} v_i(\theta_0,\alpha_{i0})\right) \\ &\qquad + \frac{1}{\pi_i(\alpha_{i0}|\theta_0)} \frac{\partial}{\partial\alpha}\Big|_{\alpha_{i0}} \mathbb{E}_{\theta_0,\alpha_{i0}}\left(-v_i^{\alpha_i}(\theta_0,\alpha)\right)^{-1} \pi_i(\alpha|\theta_0) m_i^{\alpha_i}(\theta_0,\alpha). \end{aligned}$$

The correct expression for B_M in Theorem 5 then follows because of the two identities

$$\mathbb{E}_{\theta_0,\alpha_{i0}}\left(\left[-v_i^{\alpha_i}(\theta_0,\alpha_{i0})\right]^{-1} v_i(\theta_0,\alpha_{i0})\right) = H^{-2}\mathbb{E}_{\theta_0,\alpha_{i0}}\left(v_i^{\alpha_i}(\theta_0,\alpha_{i0}) v_i(\theta_0,\alpha_{i0})\right) + o_p\left(\frac{1}{T}\right),$$

and

$$\begin{aligned} \frac{\partial}{\partial\alpha}\Big|_{\alpha_{i0}} \mathbb{E}_{\theta_0,\alpha}\left(-v_i^{\alpha_i}(\theta_0,\alpha_{i0})\right) &= \frac{\partial}{\partial\alpha}\Big|_{\alpha_{i0}} \int \left(-v_i^{\alpha_i}(\theta_0,\alpha_{i0})\right) f_i(y_i|\theta_0,\alpha) dy_i \\ &= \int \left(-v_i^{\alpha_i}(\theta_0,\alpha_{i0})\right) \frac{\partial}{\partial\alpha}\Big|_{\alpha_{i0}} f_i(y_i|\theta_0,\alpha) dy_i \\ &= -T\mathbb{E}_{\theta_0,\alpha_{i0}}\left(v_i^{\alpha_i}(\theta_0,\alpha_{i0}) v_i(\theta_0,\alpha_{i0})\right). \end{aligned}$$

[1]We are grateful to Xiaoxia Shi for pointing out the issue with the bias formula for average marginal effects.

Finally, note that Corollary 4 is unaffected, but the $[\mathbb{E}_{\theta_0,\alpha_{i0}}(-v_i^{\alpha_i}(\theta_0,\alpha))]^{-1}$ on p. 10 of the supplementary appendix should be replaced by $[\mathbb{E}_{\theta_0,\alpha}(-v_i^{\alpha_i}(\theta_0,\alpha))]^{-1}$. The expression for $\widetilde{B}_M$ at the bottom of p. 10 is correct.[2]

References

[1] Arellano, M., and S. Bonhomme (2009a): "Robust Priors in Nonlinear Panel Data Models," *Econometrica*, 77(2), 489–536.

[2] Arellano, M., and S. Bonhomme (2009b): "Supplement to 'Robust Priors in Nonlinear Panel Data Models'," *Econometrica* Supplemental Material, 77.

[2]Another inconsistency is that the formula for the prior in (36) in the static Logit model does not exactly coincide with that of the prior in (12). Nevertheless the priors in (36) and (37) are also bias-reducing. More generally, when implementing our proposed prior one should modify the denominator in (34) to account for the presence of the cross-products (for which empirical counterparts are easily constructed). We thank Ruggero Bellio and Nicola Sartori for pointing this out to us.